Spring 2012

EE 330

ENGINEERING ELECTROMAGNETICS

HW 1: 1.3, 1.7, 1.13, 1.17, 1.29

Problem 1.3 A harmonic wave traveling along a string is generated by an oscillator that completes 180 vibrations per minute. If it is observed that a given crest, or maximum, travels 300 cm in 10 s, what is the wavelength?

Solution:

$$f = \frac{180}{60} = 3 \text{ Hz.}$$

$$u_{\text{p}} = \frac{300 \text{ cm}}{10 \text{ s}} = 0.3 \text{ m/s.}$$

$$\lambda = \frac{u_{\text{p}}}{f} = \frac{0.3}{3} = 0.1 \text{ m} = 10 \text{ cm.}$$

Problem 1.7 A wave traveling along a string in the $+x$-direction is given by

$$y_1(x,t) = A\cos(\omega t - \beta x),$$

where $x = 0$ is the end of the string, which is tied rigidly to a wall, as shown in Fig. P1.7. When wave $y_1(x,t)$ arrives at the wall, a reflected wave $y_2(x,t)$ is generated. Hence, at any location on the string, the vertical displacement y_s is the sum of the incident and reflected waves:

$$y_s(x,t) = y_1(x,t) + y_2(x,t).$$

(a) Write an expression for $y_2(x,t)$, keeping in mind its direction of travel and the fact that the end of the string cannot move.

(b) Generate plots of $y_1(x,t)$, $y_2(x,t)$ and $y_s(x,t)$ versus x over the range $-2\lambda \le x \le 0$ at $\omega t = \pi/4$ and at $\omega t = \pi/2$.

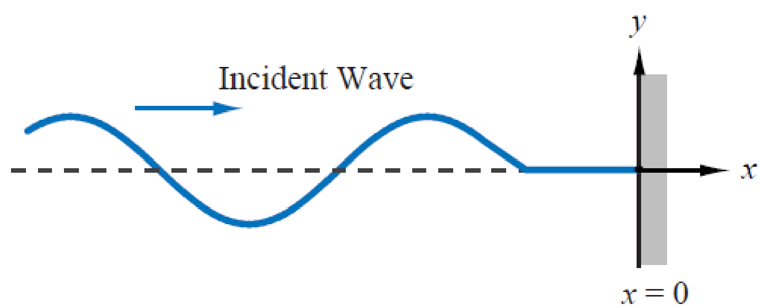

Figure P1.7: Wave on a string tied to a wall at $x = 0$ (Problem 1.7).

Solution:

(a) Since wave $y_2(x,t)$ was caused by wave $y_1(x,t)$, the two waves must have the same angular frequency ω, and since $y_2(x,t)$ is traveling on the same string as $y_1(x,t)$, the two waves must have the same phase constant β. Hence, with its direction being in the negative x-direction, $y_2(x,t)$ is given by the general form

$$y_2(x,t) = B\cos(\omega t + \beta x + \phi_0), \tag{1}$$

where B and ϕ_0 are yet-to-be-determined constants. The total displacement is

$$y_s(x,t) = y_1(x,t) + y_2(x,t) = A\cos(\omega t - \beta x) + B\cos(\omega t + \beta x + \phi_0).$$

Since the string cannot move at $x = 0$, the point at which it is attached to the wall, $y_s(0,t) = 0$ for all t. Thus,

$$y_s(0,t) = A\cos\omega t + B\cos(\omega t + \phi_0) = 0. \tag{2}$$

(i) Easy Solution: The physics of the problem suggests that a possible solution for (2) is $B = -A$ and $\phi_0 = 0$, in which case we have

$$y_2(x,t) = -A\cos(\omega t + \beta x). \tag{3}$$

(ii) Rigorous Solution: By expanding the second term in (2), we have

$$A\cos\omega t + B(\cos\omega t\cos\phi_0 - \sin\omega t\sin\phi_0) = 0,$$

or

$$(A + B\cos\phi_0)\cos\omega t - (B\sin\phi_0)\sin\omega t = 0. \tag{4}$$

This equation has to be satisfied for all values of t. At $t = 0$, it gives

$$A + B\cos\phi_0 = 0, \tag{5}$$

and at $\omega t = \pi/2$, (4) gives

$$B\sin\phi_0 = 0. \tag{6}$$

Equations (5) and (6) can be satisfied simultaneously only if

$$A = B = 0 \tag{7}$$

or

$$A = -B \quad \text{and} \quad \phi_0 = 0. \tag{8}$$

Clearly (7) is not an acceptable solution because it means that $y_1(x,t) = 0$, which is contrary to the statement of the problem. The solution given by (8) leads to (3).

(b) At $\omega t = \pi/4$,

$$y_1(x,t) = A\cos(\pi/4 - \beta x) = A\cos\left(\frac{\pi}{4} - \frac{2\pi x}{\lambda}\right),$$

$$y_2(x,t) = -A\cos(\omega t + \beta x) = -A\cos\left(\frac{\pi}{4} + \frac{2\pi x}{\lambda}\right).$$

Plots of y_1, y_2, and y_3 are shown in Fig. P1.7(b).

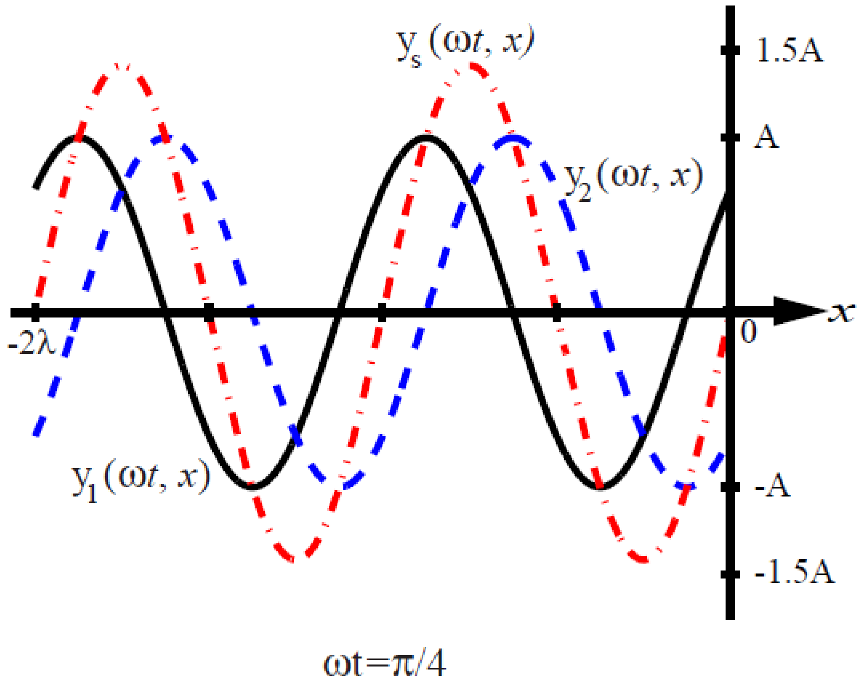

Figure P1.7: (b) Plots of y_1, y_2, and y_s versus x at $\omega t = \pi/4$.

At $\omega t = \pi/2$,

$$y_1(x,t) = A\cos(\pi/2 - \beta x) = A\sin\beta x = A\sin\frac{2\pi x}{\lambda},$$

$$y_2(x,t) = -A\cos(\pi/2 + \beta x) = A\sin\beta x = A\sin\frac{2\pi x}{\lambda}.$$

Plots of y_1, y_2, and y_3 are shown in Fig. P1.7(c).

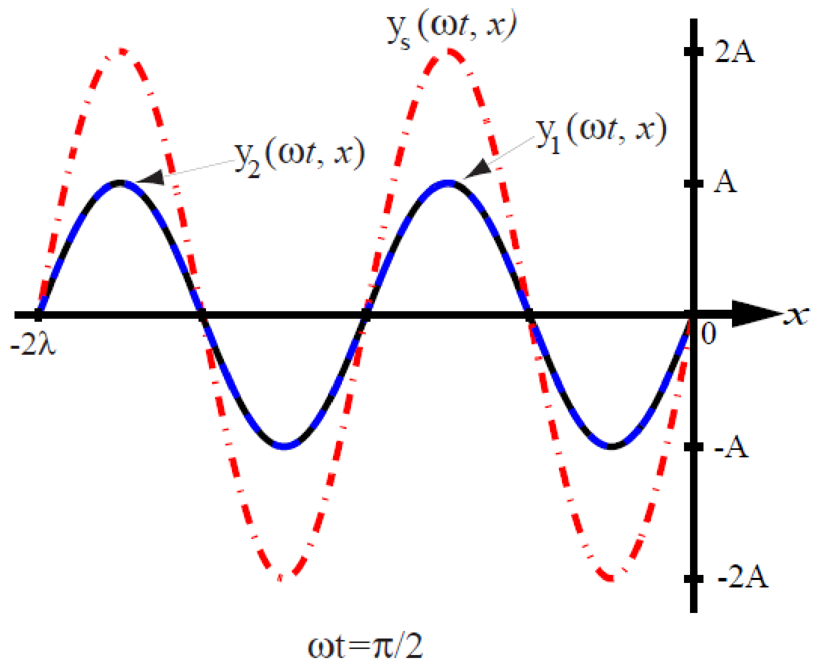

Figure P1.7: (c) Plots of y_1, y_2, and y_s versus x at $\omega t = \pi/2$.

Problem 1.13 The voltage of an electromagnetic wave traveling on a transmission line is given by $\upsilon(z,t) = 5e^{-\alpha z}\sin(4\pi \times 10^9 t - 20\pi z)$ (V), where z is the distance in meters from the generator.

(a) Find the frequency, wavelength, and phase velocity of the wave.

(b) At $z = 2$ m, the amplitude of the wave was measured to be 2 V. Find α.

Solution:

(a) This equation is similar to that of Eq. (1.28) with $\omega = 4\pi \times 10^9$ rad/s and $\beta = 20\pi$ rad/m. From Eq. (1.29a), $f = \omega/2\pi = 2 \times 10^9$ Hz $= 2$ GHz; from Eq. (1.29b), $\lambda = 2\pi/\beta = 0.1$ m. From Eq. (1.30),

$$u_p = \omega/\beta = 2 \times 10^8 \text{ m/s}.$$

(b) Using just the amplitude of the wave,

$$2 = 5e^{-\alpha 2}, \qquad \alpha = \frac{-1}{2\text{ m}} \ln\left(\frac{2}{5}\right) = 0.46 \text{ Np/m}.$$

Problem 1.17 Complex numbers z_1 and z_2 are given by

$$z_1 = 3 - j2$$
$$z_2 = -4 + j3$$

(a) Express z_1 and z_2 in polar form.
(b) Find $|z_1|$ by first applying Eq. (1.41) and then by applying Eq. (1.43).
(c) Determine the product $z_1 z_2$ in polar form.
(d) Determine the ratio z_1/z_2 in polar form.
(e) Determine z_1^3 in polar form.

Solution:

(a) Using Eq. (1.41),

$$z_1 = 3 - j2 = 3.6e^{-j33.7^\circ},$$
$$z_2 = -4 + j3 = 5e^{j143.1^\circ}.$$

(b) By Eq. (1.41) and Eq. (1.43), respectively,

$$|z_1| = |3 - j2| = \sqrt{3^2 + (-2)^2} = \sqrt{13} = 3.60,$$
$$|z_1| = \sqrt{(3 - j2)(3 + j2)} = \sqrt{13} = 3.60.$$

(c) By applying Eq. (1.47b) to the results of part (a),

$$z_1 z_2 = 3.6e^{-j33.7^\circ} \times 5e^{j143.1^\circ} = 18e^{j109.4^\circ}.$$

(d) By applying Eq. (1.48b) to the results of part (a),

$$\frac{z_1}{z_2} = \frac{3.6e^{-j33.7^\circ}}{5e^{j143.1^\circ}} = 0.72e^{-j176.8^\circ}.$$

(e) By applying Eq. (1.49) to the results of part (a),

$$z_1^3 = \left(3.6e^{-j33.7^\circ}\right)^3 = (3.6)^3 e^{-j3\times 33.7^\circ} = 46.66e^{-j101.1^\circ}.$$

Problem 1.29 The voltage source of the circuit shown in Fig. P1.29 is given by

$$v_s(t) = 25\cos(4\times 10^4 t - 45^\circ) \quad \text{(V)}.$$

Obtain an expression for $i_L(t)$, the current flowing through the inductor.

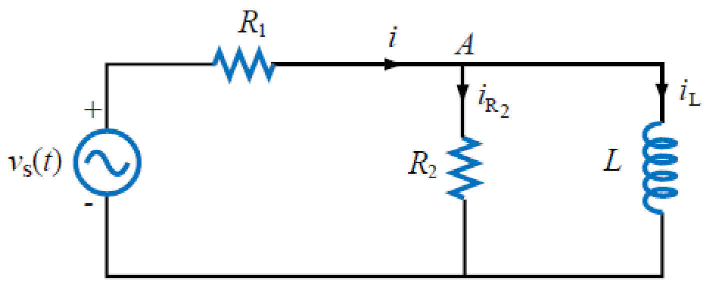

Figure P1.29:

Solution: Based on the given voltage expression, the phasor source voltage is

$$\widetilde{V}_s = 25e^{-j45^\circ} \quad \text{(V)}. \tag{9}$$

The voltage equation for the left-hand side loop is

$$R_1 i + R_2 i_{R_2} = v_s \tag{10}$$

For the right-hand loop,

$$R_2 i_{R_2} = L\frac{di_L}{dt}\ , \tag{11}$$

and at node A,

$$i = i_{R_2} + i_L. \tag{12}$$

Next, we convert Eqs. (2)–(4) into phasor form:

$$R_1\widetilde{I} + R_2\widetilde{I}_{R_2} = \widetilde{V}_s \tag{13}$$

$$R_2\widetilde{I}_{R_2} = j\omega L\widetilde{I}_L \tag{14}$$

$$\widetilde{I} = \widetilde{I}_{R_2} + \widetilde{I}_L \tag{15}$$

Upon combining (6) and (7) to solve for $\widetilde{I}_{R_2}$ in terms of $\widetilde{I}$, we have:

$$\widetilde{I}_{R_2} = \frac{j\omega L}{R_2 + j\omega L} I\ . \tag{16}$$

Substituting (8) in (5) and then solving for $\widetilde{I}$ leads to:

$$\begin{aligned}
R_1\widetilde{I} + \frac{jR_2\omega L}{R_2 + j\omega L}\widetilde{I} &= \widetilde{V}_s \\
\widetilde{I}\left(R_1 + \frac{jR_2\omega L}{R_2 + j\omega L}\right) &= \widetilde{V}_s \\
\widetilde{I}\left(\frac{R_1R_2 + jR_1\omega L + jR_2\omega L}{R_2 + j\omega L}\right) &= \widetilde{V}_s \\
\widetilde{I} = \left(\frac{R_2 + j\omega L}{R_1R_2 + j\omega L(R_1 + R_2)}\right)\widetilde{V}_s. & \qquad (17)
\end{aligned}$$

Combining (6) and (7) to solve for $\widetilde{I}_L$ in terms of $\widetilde{I}$ gives

$$\widetilde{I}_L = \frac{R_2}{R_2 + j\omega L}\widetilde{I}. \qquad (18)$$

Combining (9) and (10) leads to

$$\begin{aligned}
\widetilde{I}_L &= \left(\frac{R_2}{R_2 + j\omega L}\right)\left(\frac{R_2 + j\omega L}{R_1R_2 + j\omega L(R_1 + R_2)}\right)\widetilde{V}_s \\
&= \frac{R_2}{R_1R_2 + + j\omega L(R_1 + R_2)}\widetilde{V}_s.
\end{aligned}$$

Using (1) for $\widetilde{V}_s$ and replacing R_1, R_2, L and ω with their numerical values, we have

$$\begin{aligned}
\widetilde{I}_L &= \frac{30}{20 \times 30 + j4 \times 10^4 \times 0.4 \times 10^{-3}(20 + 30)}\, 25e^{-j45^\circ} \\
&= \frac{30 \times 25}{600 + j800}\, e^{-j45^\circ} \\
&= \frac{7.5}{6 + j8}\, e^{-j45^\circ} = \frac{7.5e^{-j45^\circ}}{10e^{j53.1^\circ}} = 0.75e^{-j98.1^\circ} \quad \text{(A)}.
\end{aligned}$$

Finally,

$$\begin{aligned}
i_L(t) &= \mathfrak{Re}[\widetilde{I}_L e^{j\omega t}] \\
&= 0.75\cos(4 \times 10^4 t - 98.1^\circ) \quad \text{(A)}.
\end{aligned}$$